Game Optimal Receding Horizon Guidance Laws and Its Equivalence to Receding Horizon Guidance Laws

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In this paper, a game optimal receding horizon guidance law (GRHG) is proposed, which does not use information of the time-to-go and target maneuvers. It is shown that by adjusting design parameters appropriately, the proposed GRHG is identical to the existing receding horizon guidance law (RHG), which can intercept the target by keeping the relative vertical separation less than the given value, within which the warhead of the missile is detonated, after the appropriately selected time in the presence of arbitrary target maneuvers and initial relative vertical separation rates between the target and missile. Through a simulation study, the performance of the GRHG is illustrated and compared with that of the existing optimal guidance law (OGL).

Key Words : Game Optimal, Receding Horizon Guidance Law, Inaccurate Time-to-Go.

1. Introduction

The proportional navigation guidance (PNG) law has been widely used for more than a few decades because it requires simple implementation and low cost. However, the PNG cannot utilize information on autopilot lags even if they are known. In order to cope with this problem of the PNG, modern guidance laws such as the optimal guidance law (OGL) and game optimal guidance law (GGL) have been investigated widely since the 1960s, despite the fact that they require more measurement information than the PNG (Gerald, 1981; Holder and Sylveser, 1990; Lin, 1994).

The OGL based on optimal control theory assumes that the target's future maneuver is completely known while the GGL based on differential game theory makes no assumption on future target maneuvers but instead considers the target's maneuver capability. In reality, it is difficult to know the future target maneuvers accurately in advance. Thus, the GGL is less sensitive to errors in estimate of current target acceleration than the OGL. Performances of the OGL and GGL are dependent on the estimation of the time-to-go, which is commonly approximated as the range between the target and missile divided by the closing velocity. Both of the OGL and GGL have a degradation of performance when the time-to-go is inaccurate or unavailable and thus, may fail to intercept the target. Although there has been some research on the accurate estimation of the time-to-go by using additional measurement information and somewhat complicated algorithms (Lee, 1985; Hull *et al*., 1991), additional measurement information can also be contaminated by noise, and the time-to-go may not be estimated accurately in the available sampling time. On the other hands, Choi and Yoon (1999) have analyzed the influence of guidance commands, generated from a

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point mass model, on the full rigid body model and proposed an attitude stabilizing strategy.

As an appropriate guidance law when the time-to-go is inaccurate or unavailable, Kim *et al.* (2000a, 2000b) have proposed the receding horizon guidance law (RHG), which does not use the time-to-go but is based on optimal control theory. It is shown that the RHG can intercept the target by keeping the relative vertical separation less than the given value, within which the warhead of the missile is detonated, after the appropriately selected time even in the presence of arbitrary target maneuvers and initial relative vertical separation rates between the target and missile. However, there are no guidance laws which do not use the time-to-go but are based on differential game theory. Thus, it will be interesting to investigate a guidance law which is based on receding horizon strategy and differential game theory.

In this paper, a game optimal receding horizon guidance law (GRHG) is proposed which does not use information of both the time-to-go and target maneuvers. It is shown that by adjusting design parameters appropriately, the proposed GRHG is identical the RHG (Kim *et al.*, 2000a). Through a simulation study, the ability of the GRHG to intercept the target is illustrated and the performance of the GRHG is compared with that of the OGL.

2. Game Optimal Guidance Law

The missile-target intercept model can be depicted as shown in Fig. 1 where the following variables are defined: σ is the line-of-sight (LOS) angle, r the missile-to-target range (it is also known as the miss distance), y the relative vertical separation, A_t the target acceleration, V_t the target velocity, A_{tn} the target normal acceleration, V_{tn} the target normal velocity, A_m the missile acceleration, V_m the missile velocity, A_{mn} the missile normal acceleration, V_{mn} the missile normal velocity, γ_m the missile flight-path angle; γ_t the target flight-path angle.

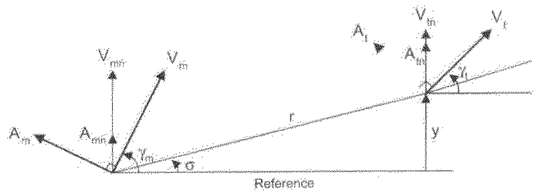

Fig. 1 Intercept geometry

The missile-target intercept model depicted in Fig. 1 is described by the nonlinear differential equations as follows (Ha *et al.*, 1990):

$$\dot{r} = V_m[\rho\cos(\theta_t) - \cos(\theta_m)] \quad \text{and} \quad r\dot{\sigma} = V_m[\rho\sin(\theta_t) - \sin(\theta_m)] \tag{1}$$

where θ_t, θ_m, and ρ are defined, respectively, by

$$\theta_t = \gamma_t - \sigma,\ \theta_m = \gamma_m - \sigma,\ \rho = V_t / V_m.$$

Here, θ_m is the angle between the missile velocity vector and the LOS, θ_t the angle between the target velocity vector and the LOS, and ρ the target-to-missile velocity ratio.

To proceed with the derivation, it is assumed that velocities V_t and V_m are constant. In this case, since $\dot{y} = V_t \sin\gamma_t - V_m \sin\gamma_m$, $A_{mn} = V_m\dot{\gamma}_m \cos\gamma_m$, and $A_{tn} = V_t\dot{\gamma}_t \cos\gamma_t$, the dynamics are expressed in terms of state variables normal to the reference:

$$\dot{X}(t) = FX(t) + BA_{mn}(t) + GA_{tn}(t) \tag{2}$$

where

$$X(t) = \begin{bmatrix} y(t) \\ \dot{y}(t) \end{bmatrix},\ F = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ -1 \end{bmatrix},\ G = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

For the dynamics (2), we consider the following differential game problem:

$$\min_{A_{mn}} \max_{A_{tn}} \int_{t_0}^{t_f} [A_{mn}^2(t) - \gamma^2 A_{tn}^2(t)]\,dt + X^T(t_f) Q_f X(t_f). \tag{3}$$

Here, γ is the target acceleration bound of differential game problem. The differential game problem described by (2) and (3) admits a unique feedback saddle-point solution, if and only if $\gamma > \hat{\gamma}^{CL}$ where the threshold value $\hat{\gamma}^{CL}$ is defined as follows (Basar, 1991):

$$\hat{\gamma}^{CL} = inf\{\gamma > 0 : K(t) \text{ does not have a conjugate point for all } t \in [t_0, t_f]\}.$$

Here, $K(t)$ is the solution of the following differential Riccati Equation:

$$-\dot{K}(t)=A^TK(t)+K(t)A \\ -K(t)[BB^T-\gamma^{-2}GG^T]K(t) \quad (4)$$

with $K(t_f)=Q_f$. Then, the unique feedback saddle point solution for $t\in[t_0, t_f)$ is given by

$$A^*_{mn}(t)=-B^TK(t)X(t) \text{ and} \\ A^*_{tn}(t)=\gamma^{-2}G^TK(t)X(t). \quad (5)$$

The solution $A^*_{mn}(t)$ is called game optimal guidance law (GGL) in this paper.

Since our purpose is to make the terminal miss distance (MD) zero (i.e., $y(t_f)=0$), we set the terminal weighting matrix Q_f as

$$Q_f=\begin{bmatrix} c & 0 \\ 0 & 0 \end{bmatrix} \text{ with } c\rightarrow\infty. \quad (6)$$

Now, we obtain an explicit form of the GGL under the condition (6). We define $K(t)$ as $K(t)=\begin{bmatrix} K_1 & K_2 \\ K_2 & K_3 \end{bmatrix}$. Then, from the Riccati Differential Equation (4), we obtain

$$\dot{K}_1=-K_2^2(1-\gamma^{-2}), \\ \dot{K}_2=-K_1-K_2K_3(1-\gamma^{-2}), \\ \dot{K}_3=-2K_2-K_3^2(1-\gamma^{-2}).$$

Thus, if $\gamma=1$, then we obtain $A^*_{mn}(t)=A^*_{tn}(t)=c[t_{go}y(t)+t^2_{go}\dot{y}(t)]$ where $t_{go}=t_f-t$. In order to obtain the explicit forms of $A^*_{mn}(t)$ and $A^*_{tn}(t)$ for $\gamma\neq1$, we change (5) to

$$A^*_{mn}(t)=[B^TP(t)/q(t)]M(t) \text{ and} \\ A^*_{tn}(t)=\gamma^{-2}[G^TP(t)/q(t)]M(t) \quad (7)$$

where $M(t)=P^T(t)X(t)$, and $P(t)$ and $q(t)$ are obtained, respectively, from

$$\dot{P}(t)=-F^TP(t) \text{ with } P(t_f)=\begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and}$$

$$q(t)=-\int_t^{t_f}[P^T(\tau)(BB^T-\gamma^{-2}GG_2^T)P(\tau)]d\tau.$$

For the system (2), $P(t)$ and $q(t)$ are found to be

$$P(t)=\begin{bmatrix} 1 \\ t_{go} \end{bmatrix} \text{ and} \\ q(t)=-\frac{1-\gamma^{-2}}{3}t^3_{go}, \text{ respectively.}$$

Therefore, the explicit forms of $A^*_{mn}(t)$ and $A^*_{tn}(t)$ can be expressed, respectively, as

$$A^*_{mn}(t)=\frac{3}{(1-\gamma^{-2})t^2_{go}}[y(t)+t_{go}\dot{y}(t)] \text{ and} \\ A^*_{tn}(t)=\frac{3\gamma^{-2}}{(1-\gamma^{-2})t^2_{go}}[y(t)+t_{go}\dot{y}(t)]. \quad (8)$$

Note that the GGL when $\gamma=\infty$ is equal to the optimal guidance law (OGL) (Holder and Sylveser, 1990; Lin, 1994) and the PNG (Ha *et al.*, 1990) when there are no autopilot lags and $N=\frac{3}{1-\gamma^{-2}}$. Since OGL also uses the navigation constant N including 3, the OGL with N is also equal to the PNG when there are no autopilot lags (Holder and Sylveser, 1990). Based on the derivation of this section, we derive game optimal receding horizon guidance law and show its equivalence to receding horizon guidance law.

3. Game Optimal Receding Horizon Guidance Law

We consider another guidance problem to minimize

$$\min_{A_{mn}}\max_{A_{tn}}\int_t^{t+T}[A^2_{mn}(\tau)-\gamma^2A^2_{tn}(t)]d\tau \\ +X^T(t+T)Q_fX(t+T). \quad (9)$$

Here, T is an adjustable design parameter which is irrespective of the final time t_f and the time-to-go t_{go}. We then propose the game optimal receding horizon guidance law (GRHG) which is obtained from the following procedure.

(a) At the present time t, the unique saddle point solutions for the problem (9), $A^*_{mn}(\tau)$ and $A^*_{tn}(\tau)$ are obtained for $\forall\tau\in[t, t+T)$.

(b) Among these controls, only the first control $A^*_{mn}(\tau)|_{\tau=t}$ is used.

(c) At the next t, the procedures (a) and (b) are repeated.

The navigation constant N is defined as $N=\frac{3}{1-\gamma^{-2}}$. Then, according to the above procedure, the proposed GRHG is given by

$$A^*_{mn}(t)=\frac{3}{(1-\gamma^{-2})T^2}[y(t)+T\dot{y}(t)] \\ =\left[\frac{N}{T^2} \quad \frac{N}{T}\right]X(t) \quad (10)$$

Thus, the proposed GRHG is identical to the RHG (Kim *et al*., 2000a) and thus, has the same property as mentioned in the following. It is noted that $N=3$ when $\gamma \to \infty$ and $N \to \infty$ when $\gamma \to 1$.

We assume that the reference is aligned with the LOS at initial time, i.e., $y(0)=0$, that the initial relative vertical separation rate is in the range of a positive constant β, i.e., $|\dot{y}(0)| \leq \beta$, that the target is arbitrarily maneuvering in the range of a positive constant α, i.e., $|A_{tn}(t)| \leq \alpha$, and that the warhead of the missile is detonated within some distance R_D by the function of a proximity fuse.

For any $t_D>0$ and $R_D>0$, the GRHG satisfies

$$|y(t)| \leqq R_D \text{ for } t \geq t_D \quad (11)$$

if the following conditions depending on γ are satisfied

$$R_D \geq \begin{cases} \dfrac{T}{\sqrt{N(N-4)}} e^{\frac{-N+\sqrt{N(N-4)}}{2T} t_D} \beta + \dfrac{T^2}{N} \alpha \text{ for } 0<\gamma<2 (\gamma \neq 1) \\ \left\{ t_D e^{-\frac{2}{T} t_D} \beta \text{ for } t_D \geq \dfrac{T}{2}, \dfrac{T}{2} e^{-1} \beta \text{ for } t_D < \dfrac{T}{2} \right\} + \dfrac{T^2}{4} \alpha \text{ for } \gamma=2 \\ \dfrac{2T}{\sqrt{N(4-N)}} e^{-\frac{N}{2T} t_D} \beta + \dfrac{4T^2}{N\sqrt{N(4-N)}} \alpha \text{ for } \gamma>2 \end{cases} \quad (12)$$

where there always exist two free parameters γ (i.e., N) and T satisfying these conditions.

This property means that the proposed GRHG can intercept the target by using γ and T, irrespective of t_{go} only if $t_D < t_f$ while with the OGL and the GGL, the missile can intercept the target when t_{go} is accurate. It should be noted that in order to intercept the target, the terminal MD has only to be within R_D.

In the next section, the performance of the proposed GRHG is compared with that of OGL when the time-to-go is inaccurate and there are no autopilot lags.

4. Simulation Example

Similarly to the simulation example of Kim *et al*. (2000a), it is assuned that V_m=Mach 2, V_t= Mach 1, $\gamma_m(0)=-1$ deg, $\gamma_t(t_0)=180$ deg, r= 5,000 ft, the missile can intercept the target if MD is below 4 ft, there is no measurement error for $y(t)$ and $\dot{y}(t)$, the future target acceleration is not known, $t_0=0$, $y(0)=0$, the missile is not on a collision course at initial time, i.e., $\dot{y}(0) \neq 0$, $A_t(t)=10g$, the missile can maneuver in the range of $|A_m(t)| \leqq 30g$ where g is acceleration of gravity, and t_{go} is computed by

$$t_{go} = -\frac{r(t)}{\dot{r}(t)}$$

where $t_{go} \approx 1.5$ sec at initial time.

From the assumptions, $|A_{tn}(t)| \leq 10\text{g}(=\alpha)$ and $|\dot{y}(0)| = |V_t \sin \gamma_t(0) - V_m \sin \gamma_m(0)| \leq 38.9$ ft/s$(=\beta)$.

Then, the GRHG is designed to keep the lateral MD less than 4 $ft(=R_D)$ from t_D to the intercept time. From Eq. (11), we know that $|y(t)| \leq 3.35$ ft for $t \geq t_D$ sec with the pair ($t_D=0.5$, $\gamma=2$, $T=0.2$ sec). For the OGL (or PNG), we set N as $N=3$.

Figure 2 shows that $|y(t)| \leq 3.35$ ft for $t \geq 0.5$ sec.

The t_{go} with estimation errors is modeled as follows (Nesline et al., 1979):

$$\hat{t}_{go} = a t_{go} + b,$$

where a is a scale factor error and b is a bias error. In the model, the t_{go} is accurate when $a=1$ and $b=0$.

The terminal MD at $t=1.5$ sec is shown when $a=0.7+0.1(k-1)$ in Fig. 3 and $b=-0.15+0.05(k-1)$ in Fig. 4 for $k=1, 2, \cdots, 7$. It is noted that the t_{go} is accurate when $k=4$. Figures 3 and 4 show terminal miss distances for each a when $b=0$ and for each b when $a=1$, respectively.

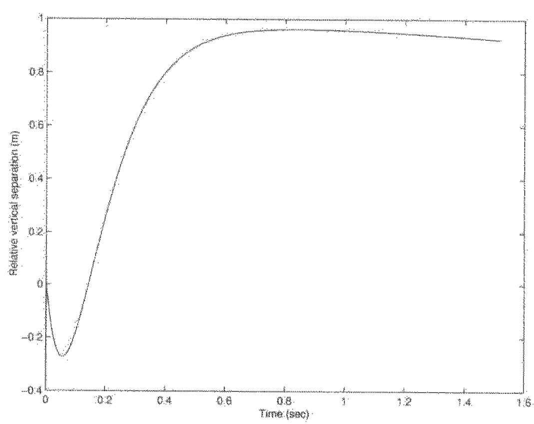

Fig. 2 Relative vertical separation y of GRHG

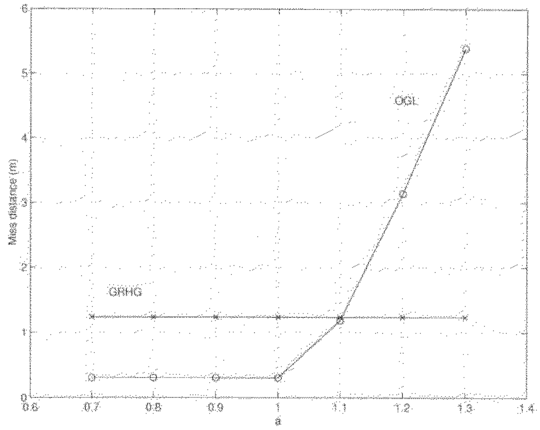

Fig. 3 The effect of a scale factor error in t_{go}

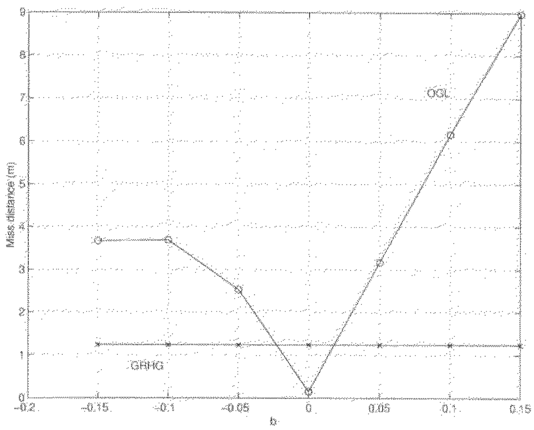

Fig. 4 The effect of a bias error in t_{go}

As shown in Figs. 3 and 4, as errors in t_{go} increase with the performance of the OGL degrades rapidly and the missile with the OGL fails to intercept the target, whereas the performance of the GRHG is not affected by t_{go}. The failure to intercept the target occurs because the OGL is saturated when the time-to-go has errors. The saturated feedback of the OGL increases with the terminal MD and then, causes a failure to intercept the target, even if the time-to-go is accurate near the final time t_f.

5. Conclusion

In this paper, a game optimal receding horizon guidance law (GRHG) has been proposed. It is shown that the proposed GRHG is identical to the receding horizon guidance law (RHG) (Kim *et al.*, 2000a) by adjusting design parameters appropriately. The proposed GRHG can intercept the target by keeping the relative vertical separation less than the given value, within which the warhead of the missile is detonated, after the appropriately selected time even in the presence of arbitrary target maneuvers and initial relative vertical separation rates between the target and missile. Through a simulation study, the performance of the GRHG is illustrated and compared with the existing optimal guidance law (OGL).

The performances of the optimal guidance law (OGL) and the game optimal guidance law (GGL) are dependent on the estimation of the time-to-go and thus, they have a computational burden for estimating the time-to-go. For this reason, the proposed GRHG and RHG (Kim *et al.*, 2000a) can be appropriate guidance laws when the time-to-go is inaccurate or unavailable, such as when the time-to-go cannot be estimated accurately in the available sampling time.

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References

Basar, T., 1991, "A Dynamic Games Approach to Controller Design : Disturbance rejection in discrete-time," *IEEE Transaction, on Automatic Control*, Vol. 36, pp. 936~952.

Choi, J. W. and Yoon, J., 1999, "An Attitude Stability Accommodating Guidance Technique for a Launch Vehicle," *KSME International Journal*, Vol. 35, No. 2, pp. 700~708.

Gerald M. A., 1981, "Comparison of Optimal

Control and Differential Game Intercept Missile Guidance Laws," *Journal of Guidance and Control*, Vol. 4, No. 2, pp. 109~115.

Ha, I. J., Hur, J. S., Ko, M. S. and Song, T. L., 1990, "Performance Analysis of PNG Laws for Randomly Maneuvering Targets," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 26, No. 5, pp. 713~721.

Holder, E. J. and Sylveser, V. B., 1990, "An Analysis of Modern Versus Classical Homing Guidance," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 26, No. 4, pp. 599~606.

Hull, D. G., Radke, J. J. and Mack, R. E., 1991, "Time-to-Go Prediction for Homing Missiles Based on Minimum-Time Intercepts," *Journal of Guidance, Control, and Dynamics*, Vol. 14, No. 5, pp. 865~871.

Kim, K. B., Kim, M. J. and Kwon, W. H., 2000a, "Receding Horizon Guidance Laws with No Information on the Time-to-Go," *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 2, pp. 193~199.

Kim, K. B., Kim, M. J. and Choi, J. W., 2000b, "Modified Receding Horizon Guidance Law with Information on Small Accurate Time-to-Go," *IEEE Transactions on Aerospace and* Electronic Systems, Vol. 36, No. 2, pp. 725~729.

Lee, G. K. F., 1985, "Estimation of the Time-to-Go Parameter for Air-to-Air Missiles," *Journal of Guidance, Control, and Dynamics*, Vol. 8, No. 2, pp. 262~266.

Lin, C. F., 1994, *Advanced Control System Design*, Prentice Hall Series in Advanced Navigation, Guidance, and Control, and Their Applications, Prentice Hall, New Jersey.

Nesline, F. W. and Zarchan, P., 1979, "A New Look at Classical vs Modern Homing Missile Guidance," *Journal of Guidance and Control*, Vol. 4, No. 1, pp. 78~85.